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(Hrsg.)

Von Rang und Namen

Philosophical Essays in Honour of Wolfgang Spohn

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INHALT

VORWORT

Vorliegender Sammelband vereint zwanzig neue philosophische Essays zu Ehren Wolfgang Spohns.

Wolfgang Spohn ist einer der großen analytischen Philosophen unserer Zeit, mit substanziellen Beiträgen zu fast allen zentralen Themen der Theoretischen und sogar einigen der Praktischen Philosophie. Den gemeinsamen Hintergrund für viele seiner Überlegungen bildet dabei die Theorie der Rangfunktionen, die Wolfgang Spohn seit 1988 in zahlreichen Aufsätzen und schließlich in seinem Opus magnum *The Laws of Belief* (OUP 2012) entwickelt hat. Angesichts der Rolle der Rangtheorie für sein Schaffen ergab sich die Wahl des Buchtitels beinahe von selbst.

Ehrungen erhielt Wolfgang Spohn zuhauf und zu Recht. Um nur die jüngsten zu erwähnen: 2012 wurde er als bislang einziger nicht-angloamerikanischer Philosoph mit dem Lakatos Award ausgezeichnet, und 2015 erhielt er den Frege-Preis der Gesellschaft für Analytische Philosophie. Doch um es in seinen eigenen Worten zu sagen: »Honours are not important. Philosophy is.« Wir haben deshalb Weggefährten, Freunde und Schüler um philosophische Essays gebeten. Die Rückmeldung war überwältigend, sodass der resultierende Band deutlich umfangreicher wurde als ursprünglich geplant.

Die Beiträge spiegeln die Bandbreite von Wolfgang Spohns Arbeiten wider: Sie behandeln Themen aus der Erkenntnistheorie (z. B. die Theorie der Rangfunktionen, Glaubensrevision, die Natur von Wissen und Überzeugungen), der Wissenschaftstheorie (z. B. Kausalität, Induktion, Naturgesetze), der Sprachphilosophie (z. B. Bedeutungstheorie, Semantik kontrafaktischer Aussagen) und der Philosophie des Geistes (z. B. Intentionalität, Willensfreiheit) ebenso wie Fragen der Ontologie, der Logik, der Theorie der praktischen Rationalität und der Metaphilosophie. Die einzelnen Arbeiten sind aber nicht immer einfach zu kategorisieren. Manche lassen sich nicht ohne Gewalt einem der genannten Themengebiete zuordnen, andere umspannen mehrere. Wir haben die Aufsätze deshalb nicht in thematische Gruppen eingeteilt. Zur Übersicht sind den Artikeln englische Zusammenfassungen vorangestellt. Dass der Band damit den Charakter des Jahresbandes einer philosophischen Zeitschrift annimmt,

Wolfgang Freitag and Alexandra Zinke

RANKS FOR THE RIDDLE?

Spohn Conditionalization and Goodman's Paradox

ABSTRACT

The paper investigates the prospects of Spohn's ranking theory with respect to Goodman's ›New Riddle of Induction‹. Based on a novel analysis of the riddle, we show it to be an inductive extension of Hansson's puzzle (Hansson 1992; 1999). As a consequence, a solution needs to take into account the dependence relations between evidential beliefs: »grue« is unprojectible because it depends on evidence whose projection is defeated. It will be suggested that this solution can be implemented in Spohn's ranking theory, but that ranking theory is unable to provide a proper explanation of the required conditional ranks.

To Wolfgang Spohn, for his unconditional support.

The present paper explores the prospects of Wolfgang Spohn's ranking theory with respect to Goodman's ›New Riddle of Induction‹. We suggest that Goodman's riddle is Hansson's puzzle (Hansson 1992; 1999) framed in the context of inductive inference.[1] Like Hansson's original puzzle, Goodman's riddle can be resolved by taking into account the dependency relations between evidential beliefs. Our aim is to show that this solution can be implemented in ranking theory. We also claim, however, that this victory is partial at best: ranking theory does not provide an explanation of the required conditional ranks. We take this to show that, as a theory of confirmation, ranking theory is crucially incomplete.

The argument proceeds as follows. Chapter 1 presents Hansson's original puzzle, described as a problem for classical Alchourrón–Gärdenfors–Makinson (AGM) belief-revision theory, and its solution in terms of a distinction between basic and dependent beliefs. Chapter 2 provides an

[1] For a conceptual precursor to this idea, see Freitag 2015 and Freitag forthcoming.

inductive extension of Hansson's puzzle, identified as Goodman's riddle, and its solution in terms of doxastic dependence: the grue-hypothesis is not projectible, because the evidence for »grue« is dependent on the evidence for a defeated projection. We implement this solution in ranking theory in chapter 3, but end with some skeptical remarks on the explanatory success of ranking theory in chapter 4.

1. HANSSON'S PUZZLE, AGM, AND DOXASTIC DEPENDENCE

Any two of the triad of predicates F, G, and $F \leftrightarrow G$ jointly entail the third.[2] Assuming that F and G are logically independent, any two of the three predicates are pairwise logically independent: none of them individually entails any of the others. As a consequence, the falsity of, say, »a is F« shows that either »a is G« or »a is $F \leftrightarrow G$« is false, but it does not determine which one of these must be abandoned. Observations like these motivate the following puzzle, formulated by Sven Ove Hansson (1992, 89–90; 1999, 18).

Consider Ann who randomly draws n objects (of type T), $a_1, \dots, a_n$, from an urn and upon closer examination forms the following beliefs:

(p) $a_1, \dots, a_n$ are F, and

(q) $a_1, \dots, a_n$ are G.

Assume, for the sake of simplicity, that F and G are independently observable properties and that Ann has identified the a's to have these properties as a result of two independent perceptual processes.

Being logically schooled, she infers from p and q that

($p \leftrightarrow q$) $a_1, \dots, a_n$ are $F \leftrightarrow G$.[3]

Suppose that Ann is then informed that some a's are not F. She has misperceived the a's with respect to this very property. Ann cannot, on pain of inconsistency, retain both that the a's are G and that they are $F \leftrightarrow G$. But which belief should she keep, and which should she abandon?[4] Our intuitive answer is obvious. Being informed that some a's are not F, she properly sticks to the view that they are G and rejects the belief that they are $F \leftrightarrow G$.

2 We say that *a predicate A entails a predicate B* iff x's being A logically entails x's being B.

3 Here we do not distinguish between »$a_1, \dots, a_n$ are $F \leftrightarrow a_1, \dots, a_n$ are G« and the logically stronger »$a_1, \dots, a_n$ are $F \leftrightarrow G$«.

4 Of course Ann could also give up both the belief that all a's are F and the belief that all a's are G, maybe now believing of some a's that they are F and of others that they are G.

Compare, however, her doxastic cousin, Cen, whose doxastic state is identical to that of Ann, except that the predicates »G« and »$F \leftrightarrow G$« are switched. That is, Cen has investigated the n a's and, like Ann, has observed that the a's are F. He has not, however, seen that they are G; he only derives that the a's are G from the observation that they are F and the additional information, obtained from an independent and presumably very reliable source, that they are $F \leftrightarrow G$. If he were confronted with the recalcitrant information that not all a's are F, he would continue to believe that the n objects are $F \leftrightarrow G$, but abandon the belief that they are G. Ann and Cen start with the very same belief set $K_A = K_C = \{p, q, p \leftrightarrow q\}$ and are confronted with the same recalcitrant information that $\neg p$. Nevertheless they end up with conflicting views.[5] A solution to *Hansson's puzzle* provides a theoretical explanation of this outcome.[6]

Observe that the relevant difference between q and $p \leftrightarrow q$ in the individual cases is not due to any *intrinsic* difference between the corresponding beliefs, e. g., a difference with respect to the metaphysical, semantic, or syntactic features of the predicates involved. Nor does it matter whether or not »$F \leftrightarrow G$« is conceptually prior to »G«. As the Cen case indicates, the possibly simple and qualitative predicate »F« is not as such preferable to the nonqualitative and complex »$F \leftrightarrow G$«. Furthermore, the difference in revision behaviour does not depend on any prior preference for either of the conflicting hypotheses. Even if Cen had a preference for q over $p \leftrightarrow q$ prior to examining the a's, he must abandon q after finding that p is false.

Hansson's puzzle was designed specifically as a challenge for classical AGM belief-revision theory, so it is worthwhile to briefly present it in this context. The standard AGM framework assumes the objects of beliefs to be sentences of some propositional language $\mathcal{L}$. For simplicity, we assume $\mathcal{L}$ to be finite. $\mathcal{L}$ is accompanied by the classical consequence relation Cn: Cn(A) is the set of sentences following deductively from the sentences

5 Here is one of Hansson's own examples:

> Let α denote that the Liberal Party will support the proposal to subsidize the steel industry, and let β denote that Ms Smith, who is liberal MP, will vote in favour of that proposal. Abe has the basic beliefs α and β, whereas Bob has the basic beliefs α and $\alpha \leftrightarrow \beta$. Thus, their beliefs (on the belief set level) with respect to α and β are the same. Both Abe and Bob receive and accept the information that α is false, and they both revise their belief states to include the new belief that $\neg\alpha$. After that, Abe has the basic beliefs $\neg\alpha$ and β, whereas Bob has the basic beliefs $\neg\alpha$ and $\alpha \leftrightarrow \beta$. Now, their belief sets are no longer the same. Abe believes that β, whereas Bob believes that $\neg\beta$. (Hansson 1999, 20)

For further examples, see Hansson 1992, 89–90; and 1999, sec. 1.7.

6 We use the label »Hansson's puzzle« mainly for dialectic reasons. As emerges below, we do not think that it is really puzzling that Ann and Cen differ in their rational revision behaviour, and consider the response to Hansson's puzzle to be quite obvious.

in A. The beliefs of an agent are represented by her *belief set* K, which is some logically closed set of sentences of $\mathcal{L}$, i.e., $K = \mathrm{Cn}(K)$.

Assume that an agent receives some new information φ. What does her posterior belief set $K*\varphi$ look like? There are two different cases: φ is either consistent or inconsistent with the belief set K. In the first case, the agent simply adds φ to her belief set and closes under deduction. This updating behaviour is called *expansion* (»+«): $K*\varphi = K+\varphi = \mathrm{Cn}(K \cup \{\varphi\})$. If φ is inconsistent with K, there must be a *revision*. Revision by φ takes place in two steps: one first ›makes room for‹ φ and then adds φ. To employ AGM terminology, one first *contracts* by $\neg\varphi$ and then *expands* by φ: $K*\varphi = (K \div \neg\varphi) + \varphi$.[7] In order to give a full account of revision, we thus have to look closer at contraction.

There are three pretheoretic desiderata on contracted belief sets: $(K \div \varphi)$ should not imply φ, not contain any new beliefs, and not abandon beliefs unnecessarily. The notion of a remainder set captures these intuitions:[8] For a set A and a sentence φ, the *remainder set* $A \perp \varphi$ is the set of maximal subsets of A not implying φ. A *selection function* for a set A of sentences is a function γ such that for all sentences φ: (i) if $A \perp \varphi \neq \emptyset$, then $\emptyset \neq \gamma(A \perp \varphi) \subseteq A \perp \varphi$, (ii) if $A \perp \varphi = \emptyset$, then $\gamma(A \perp \varphi) = A$. The selection function thus chooses some elements of the remainder set. Finally, the classical notion of contraction, *partial-meet contraction*, is defined as follows: $K \div \varphi = \bigcup \gamma(A \perp \varphi)$.[9]

We can now apply AGM belief-revision theory to Hansson's puzzle. As already noted, both Ann and Cen have the same beliefs: $K_A = \mathrm{Cn}(\{p, q\}) = \mathrm{Cn}(\{p, p \leftrightarrow q\}) = K_C$. Furthermore, they both receive the information that $\neg p$. However, intuitively, Ann should still believe that q after revising by $\neg p$, while Cen should give up his belief in q. Thus, $K_A * \neg p \neq K_C * \neg p$. In principle, there are two ways to account for this difference in the posterior belief sets of Ann and Cen. Let us explore them successively.

One way to model the different updating behaviour of Ann and Cen is by ascribing different entrenchment relations to Ann and Cen: Ann's selection function γ_A chooses sets containing q over those containing $p \leftrightarrow q$, while Cen's selection function γ_C does the reverse. As a consequence, $q \in \bigcup \gamma_A(A \perp p)$ and $p \leftrightarrow q \notin \bigcup \gamma_A(A \perp p)$, but $q \notin \bigcup \gamma_C(A \perp p)$ and $p \leftrightarrow q \in \bigcup \gamma_C(A \perp p)$. While it works technically,[10] simply ascribing differ-

7 We here use the so-called Levi identity (see Levi 1977).
8 See Alchourrón and Makinson 1981.
9 The notion of partial-meet contraction was developed in the seminal paper Alchourrón, Gärdenfors, and Makinson 1985.
10 See Gärdenfors 1990, 39 ff, where Gärdenfors applies this solution to closely related

ent entrenchment relations to Ann and Cen does not yet provide a solution to Hansson's puzzle. Such a solution would also *explain* why Ann and Cen rationally differ. We get this explanation for free by introducing the notion of a belief base, which allows us to directly represent the dependency relations between Ann's and Cen's beliefs:[11] Ann's and Cen's identical belief sets are associated with different belief bases. That is also Hansson's own strategy.[12]

A *belief base* B is a consistent, finite set of sentences of $\mathcal{L}$.[13] Intuitively, the belief base of a subject contains her ›basic beliefs‹, i.e., those beliefs that do not doxastically depend on other beliefs. Hans Rott (2000, 389) characterizes belief bases as follows: »The elements in a belief base are basic (fundamental, explicit) beliefs. They comprise beliefs, and only beliefs, which have some kind of independent standing, i.e., which are not derived from other beliefs.« We also understand the belief base as containing those beliefs that have some kind of justificatory independence, i.e., beliefs that are not believed ›just because‹ other beliefs are held. We hesitate, however, to align the distinction between non-basic and basic beliefs with that between inferred and non-inferred beliefs. Doxastic dependence cannot be reduced to actual inference. For example, the belief $\varphi \vee \psi$ may originally have been inferred from the belief φ, but then find an independent basis in the newly acquired belief ψ.

Furthermore, we do not want to be committed to the view that there are ›absolutely basic‹ beliefs. In the discussed examples, we never consider everything an agent believes, but only concentrate on those aspects of her doxastic state that are relevant for the case. With respect to the situation described in Hansson's puzzle, we will say that Ann's belief base contains the belief that p and the belief that q. This does not necessarily mean that these beliefs form the foundation of Ann's doxastic system, but only that they play the role of basic beliefs in the very restricted scenario under consideration.

examples.
11 There are various further suggestions on how to model the notion of dependence, which also account for the interplay between dependence and belief change. However, these proposals aim at notions different from our notion of doxastic dependence. For example, Parikh 1999 and Makinson and Kourousias 2006 try to capture some notion of syntactic dependence, and Fariñas del Cerro and Herzig 1996, Hansson and Wassermann 2002, and Oveisi et al. 2014 discuss notions of purely inferential dependence. Obviously, the justification operator of Haas 2005 is not meant to capture the relation of doxastic dependence either.
12 See Hansson 1992.
13 The restriction to consistent and finite bases only serves the aim of simplicity. In principle, one can allow for inconsistent and infinite belief bases.

We will now formally implement the idea of (relatively) basic vs. dependent beliefs in AGM belief-revision theory.[14] A belief base B *represents* a belief set K iff $K = \mathrm{Cn}(B)$. A belief set may be represented by different belief bases. Though Ann and Cen have the same belief set, their belief bases differ: $B_A = \{p, q\} \neq \{p, p \leftrightarrow q\} = B_C$. To model the difference in rational updating behaviour of agents with identical belief sets, belief revision will take place on belief bases. The definitions of the operations of expansion, revision, and contraction on belief bases are straightforward.[15] Let $B * \varphi$ be the belief base resulting from the belief base B after receiving evidence φ.[16] If φ is consistent with B, then $B * \varphi = B + \varphi = B \cup \{\varphi\}$. Thus, expansion on belief bases works like expansion on belief sets, except that we do not close under consequence. If φ is inconsistent with B, then, as before, $B * \varphi = (B \div \neg\varphi) + \varphi$. The definition of partial-meet contraction on belief sets can also be transferred to belief bases: $B \div \varphi = \bigcup \gamma(B \perp \varphi)$.[17] If belief bases rather than belief sets are contracted, the AGM rules recommend giving up a belief in case we no longer accept its doxastic basis. This is known as Fuhrmann's *filtering condition*, which states, roughly, that if an agent believes that φ ›just because‹ she believes that ψ, then she should not continue believing that φ if she gives up believing ψ.[18]

Hansson's puzzle dissolves once belief revision operates on belief bases. If Ann revises her belief base $B_A = \{p, q\}$ by $\neg p$, she arrives at the posterior belief base $B_A * \neg p = \{\neg p, q\}$. If Cen revises his belief base $B_C = \{p, p \leftrightarrow q\}$ by $\neg p$, he arrives at the posterior belief base $B_C * \neg p = \{\neg p, p \leftrightarrow q\}$.[19] Ann's posterior belief set $\mathrm{Cn}(\{\neg p, q\})$ contains q, while Cen's posterior belief set $\mathrm{Cn}(\{\neg p, p \leftrightarrow q\})$ does not contain q. We arrive at the desired result that

14 For the following definitions, see, e.g., Hansson 1999, sec. 1.7.

15 For an elaborated discussion of different kinds of basic belief change, see Rott 2001, sec. 3.4.1 and ch. 5.

16 For reasons of readability, we use the same symbols for revision, expansion, and contraction, respectively, on belief sets and belief bases.

17 Contraction on belief bases does not satisfy Gärdenfors' recovery postulate (see, e.g., Hansson 1999, 71–72). As this is the most contentious basic AGM postulate, that may be seen as a further advantage of base contraction over contraction on belief sets. For a defense of the recovery postulate against prominent counterexamples, however, see Spohn 2012, sec. 11.3; and 2014.

18 Fuhrmann 1991, 184. For a similar remark, see also Makinson 1997, 475.

19 The remainder set $(\{p, q\} \perp p)$ is the singleton $\{\{q\}\}$ and the remainder set $(\{p, p \leftrightarrow q\} \perp p)$ is the singleton $\{\{p \leftrightarrow q\}\}$, so here contraction is just set-theoretic subtraction. However, this is only due to the simplicity of the example. Even if revision takes place on belief bases, the selection function does not become superfluous. Consider Ben, whose belief base B_B looks as follows: $B_B = \{p, q, p \leftrightarrow q\}$. Here, both $\{q\}$ and $\{p \leftrightarrow q\}$ are elements of $(B_B \perp p)$. The selection function has to choose between these sets: as both q and $p \leftrightarrow q$ are basic beliefs, the choice cannot be motivated by dependence relations between these beliefs.

Ann continues to believe the very belief Cen gives up, and *vice versa*. This resolves Hansson's puzzle.

2. HANSSON'S PUZZLE AND GOODMAN'S RIDDLE

Given base revision, AGM theory is perfectly suited to solve Hansson's puzzle in its original form. Let us now introduce a ›new‹ puzzle which arises from Hansson's puzzle in inductive contexts. Assume that Ann's and Cen's doxastic situations prior to further, recalcitrant information are as described above, and that our doxastic agents are not yet content with the cited beliefs alone. Knowing that there remains one more object a_{n+1} (of type T) in the urn, in whose properties they have a keen interest, Ann and Cen inductively infer, on the basis of their belief sets $K_A = K_C = \mathrm{Cn}(\{p, q, p \leftrightarrow q\})$, the following three hypotheses. (Assume that our agents entertain the inductive hypotheses only because they have the inductive evidence.)

(P) a_{n+1} is F,

(Q) a_{n+1} is G, and

($P \leftrightarrow Q$) a_{n+1} is $F \leftrightarrow G$.

Furthermore, imagine that Ann and Cen are told, and accept, that although p is true P is false: *the final object* a_{n+1} *is non-F*.[20] This new piece of information creates another instance of Hansson's puzzle, this time with respect to the three *inductive* hypotheses: given that P is known to be false, Q and $P \leftrightarrow Q$ are incompatible and cannot both be retained. Again, the question is which hypothesis to keep and which one to abandon.

Let us hasten to add that also the inductive version of Hansson's puzzle receives a pretheoretically straightforward answer.[21] For both Ann and Cen the hypothesis P is only confirmed by the inductive evidence p. Both receive and accept the new information non-P, and thus abandon P and thereby the inductive prospects of p. Now, first consider Ann, who has found out that the first a's are G independently of her identification of them being F, i.e., she has the evidence q independently of her p-evidence. Ann thus rationally keeps Q: she is justified in believing that the final object

20 In other words, the new information constitutes a defeater for the inductive hypothesis P. In the case of information that contradicts an inductive hypothesis, John Pollock (e.g., 1984, 424) speaks of a »rebutting defeater«. We will here focus on rebutting defeaters exclusively.

21 For the following solution, compare also Freitag 2015 and Freitag forthcoming.

is G. As Ann believes the final object to be non-F and to be G, she also believes it to be non-$(F \leftrightarrow G)$ and thus abandons $P \leftrightarrow Q$. Cen, by contrast, has perfectly independent evidence $p \leftrightarrow q$ for the hypothesis $P \leftrightarrow Q$, but only dependent q-evidence. He has obtained $p \leftrightarrow q$ from a source that is independent of the source for p, but has only derived q from $p \leftrightarrow q$ and p jointly. When he abandons P, he keeps $P \leftrightarrow Q$, but rejects Q. The challenge is again to provide a theoretical justification for Ann's and Cen's different rational updating behaviour.

Obviously, doxastic dependence must be brought to bear, because this is the only dimension with respect to which the two situations differ. Ann and Cen agree in everything, except the dependence relations between their shared evidential beliefs. Recall that for Ann $p \leftrightarrow q$ depends on p, while q is independent thereof. For Cen, q, but not $p \leftrightarrow q$, depends on p. However, the inductive context yields the consequence that doxastic dependence and recalcitrant information appear not to be aligned: doxastic dependence between the evidential beliefs p, q, and $p \leftrightarrow q$ does not in any straightforward way interact with the information that the inductive hypothesis P is false. To remedy the case, we suggest that in Ann's case not only does $p \leftrightarrow q$ depend on p, but also $P \leftrightarrow Q$ on P. In the evidential situation Ann finds herself in, she would not be justified in believing $P \leftrightarrow Q$ if she were not justified in believing P, too. Analogous considerations allow for the conclusion that, in Cen's case, Q is doxastically dependent on P. More generally, we propose that relations of doxastic (in)dependence operative on the level of evidential beliefs are mirrored by analogous relations of doxastic (in)dependence on the level of the inductive hypotheses. That is, we propose the following principle: Hypothesis Φ doxastically depends on hypothesis Ψ if and only if evidence φ doxastically depends on evidence ψ, too (assuming that the hypotheses Φ and Ψ are only believed on the basis of the (enumerative) inductive evidence φ and ψ, respectively). Relations of doxastic (in)dependence are preserved under enumerative induction.

This principle yields a straightforward explanation for our cases. Given the information that P is false, beliefs which are doxastically dependent on P are abandoned, and independent beliefs are retained. Ann concludes that Q, while Cen infers $P \leftrightarrow Q$. That Ann and Cen arrive at conflicting opinions, although they start with exactly the same beliefs, apply the very same rules for enumerative induction, and are confronted with identical additional information, is again due to the fact that their doxastic states significantly differ with respect to the doxastic dependences between their beliefs. Given the mentioned principle, the inductive version of Hansson's puzzle reduces to Hansson's original and receives a simple solution in terms of doxastic dependence alone.

Let us now turn to Goodman's riddle, as presented in the famous passage from *Fact, Fiction, and Forecast*:

> Suppose that all emeralds examined before a certain time t are green. At time t, then, our observations support the hypothesis that all emeralds are green. ... Our evidence statements assert that emerald a is green, that emerald b is green, and so on; and each confirms the general hypothesis that all emeralds are green. ... Now let me introduce another predicate less familiar than »green«. It is the predicate »grue« and it applies to all things examined before t just in case they are green but to other things just in case they are blue. Then at time t we have, for each evidence statement asserting that a given emerald is green, a parallel evidence statement that that emerald is grue. And the statements that emerald a is grue, that emerald b is grue, and so on, will each confirm the general hypothesis that all emeralds are grue. Thus ... the prediction that all emeralds subsequently examined will be green and the prediction that all will be grue are alike confirmed by evidence statements describing the same observations. But if an emerald subsequently examined is grue, it is blue and hence not green. (Goodman 1983 [1965], 73–74)

We know that all emeralds examined before t are green and hence deduce that they are grue. Yet, although the hypotheses that future emeralds are green and that they are grue both enjoy perfect inductive support by positive instances, we cannot maintain both of them. Unexamined grue emeralds are blue. The inductive hypothesis that all emeralds are green conflicts with the hypothesis that all emeralds are grue.

Over the years Goodman's riddle has been subject to so many misrepresentations and misunderstandings – no doubt partly due to Goodman's own analysis – that it is worthwhile to emphasize the following. Goodman's claim that the green- and the grue-hypothesis »are alike confirmed by evidence statements describing the same observations« misleadingly suggests that the green-evidence alone confirms both the green- and the grue-hypothesis. Yet, obviously, we cannot tell whether emeralds are grue by considering their color alone: we must know whether they have been examined before t, too. Only jointly do the green- and the examined-evidence support the grue-hypothesis. Furthermore, the green- and the grue-hypothesis conflict only for unexamined emeralds. Our predictions that the next emerald is green and that it is grue are jointly unacceptable only on the condition that we know or believe that it is as yet unexamined.

To clearly portray the structure of the grue-riddle, let F be the property of being examined before t, and let G be the property of being green. »Grue« is then defined as the predicate »F and G, or non-F and non-G«, which is logically equivalent to »$F \leftrightarrow G$«. (This definition is a slight simplification of

Goodman's original stipulation obtained by substituting »non-green« for »blue«.[22]) After having examined the first n emeralds, we believe that

(p) $a_1, \ldots, a_n$ are F (examined), and

(q) $a_1, \ldots, a_n$ are G (green).

Being logically schooled, we infer that

($p \leftrightarrow q$) $a_1, \ldots, a_n$ are $F \leftrightarrow G$ (grue).

The green-hypothesis Q and the grue-hypothesis $P \leftrightarrow Q$ are perfectly confirmed by the inductive evidence, but their joint projection is defeated by the background knowledge that a_{n+1} is unexamined, i.e., that P is false. Goodman's riddle is Hansson's puzzle in an inductive context and thus receives the same solution.

In the situation Goodman describes, which is entirely analogous to that of Ann, the grue-evidence $p \leftrightarrow q$ doxastically depends on the examined-evidence p. Given that doxastic (in)dependence is preserved under enumerative induction, the grue-hypothesis $P \leftrightarrow Q$ doxastically depends on the examined-hypothesis P, of which the green-hypothesis Q, however, is doxastically independent. As we know that P is false – the final emerald has not been examined – we also abandon $P \leftrightarrow Q$, the belief that it is grue, although we can retain Q, the independent belief that it is green. Goodman's riddle, unsolvable if we consider the content of our doxastic attitudes alone, receives a straightforward answer based on doxastic dependence. Importantly, the preference for the green-hypothesis over the grue-hypothesis is merely due to the fact that the evidential scenario Goodman describes is analogous to that of Ann. If we were in Cen's evidential situation, the grue-hypothesis would prevail and the green-hypothesis would be forsaken. Of course, given the particular choice of predicates, Cen's scenario will be very rare, perhaps even impossible, which may explain why we are so ill-disposed towards »grue«. But this should not blind us to the the fact that *if* we were in Cen's scenario, we would rationally prefer »grue« to »green«.

3. RANKS FOR THE RIDDLE

With regard to the inductive version of Hansson's puzzle, alias the grue-paradox, Ann and Cen have, prior to the defeating belief that non-P, the same beliefs. After revision by non-P, their belief sets differ. In this case,

[22] For a defense of this simplification, see Fitelson 2008, 3.

the solution cannot be implemented in standard AGM theory by referring to base-revision: the hypotheses do not qualify as basic beliefs and are thus not elements of the belief base. More precisely, it is not the case that Ann's base contains the hypotheses P and Q, while Cen's base contains P and $P \leftrightarrow Q$. Yet its failure to solve Goodman's riddle is not a defect of AGM belief-revision theory: AGM is not meant to deal with cases of (enumerative) induction.

Spohn's theory of ranking functions cannot be excused the same way, as it promises to provide a full account of the dynamic laws of belief. The problem of induction is its very center (Spohn 2012, 3), and a proper reaction to Goodman's riddle is crucial for its success. In his own recent discussions of the riddle, Spohn expresses the view that the grue-hypothesis is indeed confirmed,[23] though it must be abandoned because its »apriori impact ... is so terribly low that it will stay low even after many confirmations« (Spohn 2016, sec. 6). Although both competing hypotheses are equally confirmed, we will end up believing the green-hypothesis rather than the grue-hypothesis, because the latter remains strongly disbelieved (even though a bit less than before).[24]

Spohn does not consider this explanation to be entirely satisfactory, since the strong prior disbelief in the grue-hypothesis remains unexplained: »Impacts are just subjectively chosen, and our choice is clear, whereas the defenders of the grue-hypothesis choose entirely different initial impacts« (Spohn 2016, sec. 6). For this reason, he admits, »Goodman's new riddle evaporates only partially« (Spohn 2016, sec. 6). Yet, if his approach is the right one, he need not be too concerned. As ranking theory only delivers the static and dynamic laws of belief, it does not, and should not, not tell you what to believe unconditionally. Initial ranks and impacts remain unexplained by ranking theory. This part of the story must come from somewhere else. Maybe, so Spohn might argue, the grue-hypothesis falters because the predicate »grue« is unnatural, positional, or not embedded in existing inductive practices. For Spohn, Goodman's riddle is not properly speaking a problem for epistemology in the sense of being amenable to normative epistemological considerations. This way he is able to protect ranking theory from the charge of being crucially incomplete.

[23] »[C]razy hypotheses like ›all emeralds are grue‹ are ... just as well confirmed as plausible laws like ›all emeralds are green‹« (Spohn 2016, sec. 6).

[24] We here severely abbreviate the story, as we have omitted a discussion of the role of laws. Spohn discusses enumerative induction only as applying to (potential) laws, which are not understood as propositions, but, roughly put, as ›inference tickets‹ coming with an apriori impact. Confirmation of a law is understood as an increase of its impact. The potential law »All emeralds are grue« has a very low impact, even after it has been confirmed by positive instances. (See Spohn 2005; 2012, ch. 12; and 2016, for proper presentations.)

As we disagree with Spohn's analysis of Goodman's riddle, we think that he does not even provide a partial solution to it. We deny that the green- and the grue-hypothesis are both confirmed. In the evidential situation Goodman describes, there is no undefeated confirmation of the grue-hypothesis. Moreover, the fact that we prefer »green« over »grue« in a case of conflict cannot be explained by reference to any extra-normative preferences prior to our examination of the samples, but is solely due to the dependence relations between our evidential beliefs. If we were not in Ann's, but in Cen's evidential scenario, we would rationally prefer »grue« to »green«. Spohn's reaction to Goodman's paradox is based on a misconception of the challenge posed by the riddle.

It seems, however, that ranking theory can do better. Our analysis of Goodman's riddle allows for an implementation in ranking theory. To show this, we will first introduce some of the basic notions of ranking theory and provide a ranking-theoretic solution to Hansson's original puzzle. Then we turn to Goodman's riddle.

Ranking theory works with a Boolean algebra $\mathcal{A}$ of propositions over a space W of possible worlds. The basic notion of a negative ranking function is introduced as follows:[25]

Definition 1. *κ is a* negative ranking function *for $\mathcal{A}$ iff κ is a function from $\mathcal{A}$ into $\mathbb{N}^+ = \mathbb{N} \cup \{\infty\}$ such that for all $A, B \in \mathcal{A}$:*

(*i*) $\kappa(W) = 0$,

(*ii*) $\kappa(\emptyset) = \infty$, *and*

(*iii*) $\kappa(A \cup B) = \min(\{\kappa(A), \kappa(B)\})$.

A negative ranking function expresses degrees of disbelief. An agent is said to *believe* a proposition A just in case she disbelieves $\bar{A}$: A is believed iff $\kappa(\bar{A}) > 0$. (An agent may be undecided about A. Then we have $\kappa(A) = \kappa(\bar{A}) = 0$.) Negative ranking functions thus model both plain belief and degrees of (dis)belief. To provide for the dynamic laws of belief, we first introduce the notion of a *negative conditional rank*:

Definition 2. *The* negative conditional rank $\kappa(B \mid A)$ *of $B \in \mathcal{A}$ conditional on $A \in \mathcal{A}$ (with $\kappa(A) < \infty$) is defined as follows:* $\kappa(B \mid A) = \kappa(B \cup A) - \kappa(A)$.

Ranking theory has different kinds of conditionalization. We here present only *result-oriented conditionalization* ($A \rightarrow n$*-conditionalization*), also known

[25] For the following definitions, see, e.g., Spohn 2012, ch. 5.

as *Spohn conditionalization*.[26] Spohn conditionalization tells you how to revise your rank for B given that you ascribe a certain rank n to a proposition A.

Definition 3. *Let κ be a negative ranking function for $\mathcal{A}$ and $A \in \mathcal{A}$ such that $\kappa(A)$, $\kappa(\bar{A}) < \infty$, and $n \in \mathbb{N}^+$. The* $A \rightarrow n$-conditionalization $\kappa_{A \rightarrow n}$ *of κ is defined by* $\kappa_{A \rightarrow n}(B) = \min\{\kappa(B \mid A), \kappa(B \mid \bar{A}) + n\}$.

The operations of expansion, contraction, and revision in AGM theory can be understood as special cases of Spohn conditionalization.[27]

Conditional ranks are not only central for the rules of belief revision, but also for Spohn's account of *reasons* and of one proposition's being *dependent* on another. Basically, A is a reason for B iff the belief in A strengthens the belief in B:

Definition 4. *Let κ be a negative ranking function for $\mathcal{A}$ and $A, B \in \mathcal{A}$. Then A is a* reason for *B w.r.t. κ iff $\kappa(\bar{B} \mid A) > \kappa(\bar{B} \mid \bar{A})$ or $\kappa(B \mid A) < \kappa(B \mid \bar{A})$. A is a* reason against *B w.r.t. κ iff A is a reason for $\bar{B}$ w.r.t. κ.*

Definition 5. *A is* relevant to *B or* dependent on *B w.r.t. κ iff A is a reason for or against B w.r.t. κ.*

The ranking-theoretic solution to Hansson's puzzle must obviously differ from our favourite solution within AGM belief-revision theory. In contradistinction to the revision rules of AGM, which operate on sentences, ranking functions take the objects of belief to be propositions, modelled as sets of possible worlds. There is hence no way, not even conceptually, to distinguish between logically equivalent beliefs or sets of beliefs. As Ann's and Cen's belief bases are logically equivalent, they cannot be distinguished within ranking theory. The ranking-theoretic solution to Hansson's puzzle will thus not employ the notion of a belief base, but solve the puzzle by ascribing different conditional ranks to Ann and Cen, respectively.[28]

First, reconsider Ann, who basically believes that p and that q.[29] Let

[26] Another type of conditionalization is *evidence-oriented* conditionalization ($A \uparrow n$*-conditionalization*), originally proposed by Shenoy (1991), which says how to revise the rank of B given that the rank of A improves by n.

[27] See, e.g., Spohn 2012, sec. 5.5; and 2014, 102–103.

[28] This type of solution has been suggested by Spohn (personal communication).

[29] Although ranking functions operate on propositions, i.e., sets of possible worlds, we here represent the objects of belief in the language of propositional logic. This allows for a straightforward application of ranking theory to Goodman's riddle, where, e.g., the grue-hypothesis has the form of a material biconditional. We take that to be unproblematic, as sentences of propositional logic can be ›translated‹ into propositions in the obvious way.

$\kappa_A(\neg p) = \kappa_A(\neg q) = m$ (with $m \in \mathbb{N}$, $m > 0$),[30] which reflects that Ann believes p and q to the same degree. To solve Hansson's puzzle, we need not only model what is believed, but also the dependency relations between the beliefs: Ann's q-belief is doxastically independent of her p-belief. We model the doxastic independence of q from p by the ranking-theoretic notion of independence: $\kappa_A(q \mid p) = \kappa_A(q \mid \neg p)$ and $\kappa_A(\neg q \mid p) = \kappa_A(\neg q \mid \neg p)$. From the ranks of $\neg p$ and of $\neg q$ and the independence of q from p, we can derive that $\kappa_A(\neg(p \leftrightarrow q)) = m$ and that $p \leftrightarrow q$ is ranking-theoretically dependent on p: $\kappa_A(\neg(p \leftrightarrow q) \mid p) = m > 0 = \kappa_A(\neg(p \leftrightarrow q) \mid \neg p)$ and $\kappa_A(p \leftrightarrow q \mid p) = 0 < m = \kappa_A(p \leftrightarrow q \mid \neg p)$.[31,32] Now assume that Ann receives the information that p is false, which makes her assign the new rank k to the proposition p (with $k \in \mathbb{N}$, $k > 0$). By Spohn conditionalization we get: $\kappa_{A\,\neg p \to k}(p) = k$, $\kappa_{A\,\neg p \to k}(\neg q) = m$, $\kappa_{A\,\neg p \to k}(p \leftrightarrow q) = \min\{k, m\}$.[33] Thus, after receiving the information that non-p, Ann continues to believe that q, but now disbelieves that $p \leftrightarrow q$, and of course she disbelieves that p. This is the desired result.

Cen basically believes p and $p \leftrightarrow q$ to the degree m, that is, $\kappa_C(\neg p) = \kappa_C(\neg(p \leftrightarrow q)) = m$. However, Cen's $p \leftrightarrow q$-belief is independent of his p-belief. It follows that $\kappa_C(q) = m$ and that Cen's q-belief is dependent on his p-belief: $\kappa_C(\neg q \mid p) = m > 0 = \kappa_C(\neg q \mid \neg p)$ and $\kappa_C(p \leftrightarrow q \mid p) = 0 < m = \kappa_C(p \leftrightarrow q \mid \neg p)$.[34] If Cen now learns that non-p, thereby assigning the new rank k to the proposition p, his posterior ranking function looks as follows:

[30] The natural number m, as well as n, k, and j used below, is chosen arbitrarily and its exact value does not matter here.

[31] The independence of p from q and $\kappa_A(\neg p) = \kappa_A(\neg q) = m$ determine that Ann's ranking function is as follows:

κ_A	p	$\neg p$
q	0	m
$\neg q$	m	$2m$

We have $\kappa_A(p \wedge q) = 0$, because $\kappa_A(\neg p \vee \neg q) = \min\{\kappa_A(\neg p), \kappa_A(\neg q)\} = m > 0$. Furthermore, $\kappa_A(q \wedge \neg p) = m$, because $0 = \kappa_A(q \mid p) = \kappa_A(q \mid \neg p)$ and $\kappa_A(\neg p) = m$. Next, $\kappa_A(p \wedge \neg q) = m$, because $0 = \kappa_A(p \mid q) = \kappa_A(p \mid \neg q)$ (symmetry of independence; see, e.g., Spohn 2012, 110) and $\kappa_A(\neg q) = m$. Finally, $\kappa_A(\neg p \wedge \neg q) = 2m$, because $m = \kappa_A(\neg p \mid q) = \kappa_A(\neg p \mid \neg q)$ (symmetry of independence) and $\kappa_A(\neg q) = m$. It follows trivially that $\kappa_A(\neg(p \leftrightarrow q)) = \min\{\kappa_A(\neg p \wedge q), \kappa_A(p \wedge \neg q)\} = m$, that $\kappa_A(\neg(p \leftrightarrow q) \mid p) = m > 0 = \kappa_A(\neg(p \leftrightarrow q) \mid \neg p)$, and that $\kappa_A(p \leftrightarrow q \mid p) = 0 < m = \kappa_A(p \leftrightarrow q \mid \neg p)$.

[32] With respect to Ann's and Cen's cases, it is the intuitively desired result that one of the three propositions p, q, and $p \leftrightarrow q$ be dependent on the other two. However, we can presumably imagine a situation in which all three propositions p, q, and $p \leftrightarrow q$ are mutually doxastically independent. (Compare Ben's situation of fn. 19 and our discussion in sec. 4.)

[33] This is because $\kappa_{A\,\neg p \to k}(p) = \min\{\kappa_A(p \mid \neg p), \kappa_A(p \mid p) + k\} = k$ and $\kappa_{A\,\neg p \to k}(\neg q) = \min\{\kappa_A(\neg q \mid \neg p), \kappa_A(\neg q \mid p) + k\} = \min\{m, m + k\} = m$ and lastly $\kappa_{A\,\neg p \to k}(p \leftrightarrow q) = \min\{\kappa_A(p \leftrightarrow q \mid \neg p), \kappa_A(p \leftrightarrow q \mid p) + k\} = \min\{m, k\}$.

[34] The reasoning is exactly analogous to Ann's case (see fn. 31); just replace q with $p \leftrightarrow q$.

$\kappa_{C\,\neg p \to k}(p) = k$, $\kappa_{C\,\neg p \to k}(\neg(p \leftrightarrow q)) = m$, $\kappa_{C\,\neg p \to k}(q) = \min\{k, m\}$. Thus, after receiving the information that p is false, Cen continues to believe that $p \leftrightarrow q$, but now disbelieves that q, and of course he disbelieves that p. By modelling doxastic dependence via ranking-theoretic dependence, we can provide a straightforward solution to Hansson's puzzle within ranking theory. How does ranking theory fare with respect to Goodman's riddle?

Let's first consider Ann's situation, i.e., the specific evidential situation Goodman describes. Ann has the evidential beliefs p, q, and $p \leftrightarrow q$. She takes the evidence propositions to yield inductive support for the corresponding hypotheses: $\kappa_A(\neg P \mid p) = \kappa_A(\neg Q \mid q) = \kappa_A(\neg(P \leftrightarrow Q) \mid p \leftrightarrow q) = j$ (with $j \in \mathbb{N}$, $j > 0$).[35] Thus, posterior to the inductive inference, Ann's ranking function with respect to the hypotheses looks as follows: $\kappa_A(\neg P) = \kappa_A(\neg Q) = \kappa_A(\neg(P \leftrightarrow Q)) = j$. We have proposed that doxastic dependence is preserved under enumerative induction: as p and q are doxastically independent, so are P and Q, and as $p \leftrightarrow q$ doxastically depends on p, so $P \leftrightarrow Q$ doxastically depends on P. Now assume that Ann receives the information that P is false, therefore assigning the new rank k to P.[36] This determines Ann's posterior ranking function with respect to P, Q, and $P \leftrightarrow Q$. By Spohn conditionalization we get: $\kappa_{A\,\neg P \to k}(P) = k$, $\kappa_{A\,\neg P \to k}(\neg Q) = j$, and $\kappa_{A\,\neg P \to k}(P \leftrightarrow Q) = \min\{j, k\}$.[37] Thus, after having received the information that non-P, Ann continues to believe that Q, but now disbelieves that $P \leftrightarrow Q$, and of course she disbelieves that P. For Goodman's riddle Cen-style, i.e., a situation in which the inductive evidences p and $p \leftrightarrow q$ are doxastically independent, ranking theory yields the same results, only with Q and $P \leftrightarrow Q$ switched. Thus, after receiving the information that non-P, Cen continues to believe that $P \leftrightarrow Q$, but now disbelieves that Q, and of course he disbelieves that P.[38]

[35] Plausibly, $j \leq m$, as we assume that our doxastic subjects believe the hypotheses only on the basis of the corresponding inductive evidence.

[36] In representing Goodman's paradox as Hansson's puzzle in a context of inductive inference, we portray it as a case of belief revision, i.e., as a case in which a given set of beliefs has to be revised on the basis of new information. But of course Ann never really comes to believe that Q, i.e., that the next emerald is sampled, because she never performs the inductive inference from the sampled-evidence to the sampled-hypothesis. So Goodman's riddle is not, strictly speaking, a case of belief revision. But note that we do not intend to provide a psychologically or procedurally adequate description of the case, but aim at displaying its logical structure.

[37] $\kappa_{A\,\neg P \to k}(P) = \min\{\kappa_A(P \mid \neg P), \kappa_A(P \mid P) + k\} = k$ and $\kappa_{A\,\neg P \to k}(\neg Q) = \min\{\kappa_A(\neg Q \mid \neg P), \kappa_A(\neg Q \mid P) + k\} = \min\{j, j + k\} = j$ and $\kappa_{A\,\neg P \to k}(P \leftrightarrow Q) = \min\{\kappa_A(P \leftrightarrow Q \mid P), \kappa_A(P \leftrightarrow Q \mid P) + k\} = \min\{j, k\}$.

[38] Note that this ranking-theoretic solution strategy only refers to the qualitative ordering of the hypotheses, not to their specific ranks. One can thus translate the solution into one for AGM-style belief revision.

If we understand the doxastic dependency relations between the hypotheses in terms of ranking-theoretic dependence, the laws of ranking theory deliver the desired results for our two cases: Ann should believe that Q, i.e., that the next object is green, while Cen should believe that $P \leftrightarrow Q$, i.e., that the next object is grue. This reflects the fact that the diverging updating behaviour of Ann and Cen is not due to a difference in prior preferences. It is not that Ann has chosen green-favouring a priori impacts, while Cen is a fan of ›crazy hypotheses‹. Ann differs from Cen only with respect to matters of ranking-theoretic dependence, due to the doxastic state she finds herself in. It seems, therefore, that we have found an alternative, and better, way of implementing Goodman's riddle in a ranking-theoretic framework.

4. A RIDDLE FOR RANKS

Given our analysis of Goodman's riddle, Ann's and Cen's divergent updating behaviours are due to a difference in the dependence relations between their beliefs. By modelling doxastic dependence via ranking-theoretic dependence, this solution can be implemented into ranking theory. Like Spohn's original response to the riddle, this solution is only partial: the laws of ranking theory do not determine that Ann's and Cen's conditional ranks for the hypotheses differ in the required way even given all other information about Ann's and Cen's doxastic states. We lack a ranking-theoretic justification of the preservation of doxastic (in)dependence under (enumerative) induction.

Our analysis of Goodman's riddle can be implemented in ranking theory, but there is again no genuine ranking-theoretic explanation of our preference for the green-hypothesis. This time, however, the explanatory gap constitutes a serious problem for the theory, because the preference for »green« over »grue« can no longer be deferred to prior subjective choices. No matter which extra-normative preferences Ann and Cen may have with respect to »green« and »grue«, their posterior beliefs are normatively determined by the doxastic dependency relations between their evidential beliefs. Our preference for the green-hypothesis is based on the interplay between doxastic dependence and inductive confirmation alone. It therefore concerns the normative rules for the dynamics of belief. If ranking theory aims at providing a complete theory of the dynamics of belief, it must fully explain our preference for the green-hypothesis. That it is unable to do so is in our view a major lacuna in the suggested ranking-theoretic implementation of the solution to Goodman's riddle.[39]

39 We think, by the way, that ranking theory's big sister, probability theory, does not fare any better.

We have tentatively modelled doxastic dependence in terms of ranking-theoretic dependence, i.e., in terms of (positive) relevance. Given this analysis, ranking theory can model, but not explain, our preference for the green-hypothesis. This seems to indicate that the ranking-theoretic notion of (in)dependence is unable to adequately model the intuitive notion of doxastic (in)dependence figuring in Hansson's puzzle and Goodman's paradox. Our suspicion is confirmed by the fact that there are possible situations in which all three propositions p, q, and $p \leftrightarrow q$ are mutually doxastically independent. For example, Ben might read three independent reports, one saying that p, another that q, and yet another that $p \leftrightarrow q$, thus coming to believe each of p, q, and $p \leftrightarrow q$ independently of the other propositions. However, this is impossible with respect to the ranking-theoretic notion of independence, as the independence of two of the triad of propositions implies the dependence of the third.

Perhaps it is possible to arrive at a more satisfactory result by giving another ranking-theoretic account of doxastic dependence, which takes care of Ben's case and also yields the desired outcomes regarding the interplay of dependence and induction. This would result in a different ranking-theoretic implementation, not only of Goodman's riddle, but also of Hansson's puzzle. However, we are not overly optimistic that doxastic dependence can be fully captured within ranking theory.[40] Of course this question can only be settled once we have a better informal grip on the notion of doxastic dependence.[41] Here we have only vaguely suggested that B doxastically depends on A iff B is believed ›only because‹ A is believed, but did not further analyze the »only because«. Indubitably there is more work lying ahead of us here. Yet, whatever an informal explication of the relation of doxastic dependence will look like in detail, doxastic dependence appears to be an asymmetric relation and therefore alien to the coherentistic spirit of the ranking-theoretic notion of a reason. Spohn himself seems to be unsure whether the ranking-theoretic dependence relations can model the »only because«, but considers this rather unimportant: »[Ranking theory] embodies justificatory relations; whether it does so in a generally acceptable way, and whether it can specifically explicate the

40 Maybe the ranking-theoretic notion of a *necessary reason* comes closer to capturing the pretheoretic notion of doxastic dependence. However, even if we model doxastic dependence by the necessary-reason relation, the relevant dependencies between the hypotheses are not implied by ranking theory. (See Spohn 2012, 107, for the ranking-theoretic definition of a necessary reason.)

41 Spohn (2012, 476–477) lists four conceptions of reasons: the positive-relevance conception, the deductive conception, the computational conception, and the causal conception. The relation of doxastic dependence does not appear to fall under any of them.

›only because‹, does not really matter« (Spohn 2012, 241).[42] We obviously disagree with the last remark: it does matter whether the »only because« can be captured in ranking-theoretic terms, as it matters whether ranking theory has sufficient expressive power to adequately address Goodman's riddle.

Our aim in this paper was to reveal Goodman's riddle as a thoroughly epistemological puzzle requiring an answer in terms of epistemic normativity. Goodman's paradox is resolved by recourse to epistemic context, in particular by reference to doxastic dependence. It thereby exemplifies Spohn's credo that »many of the philosophically most interesting notions are overtly or covertly epistemological« (Spohn 1988, 105). If the presented solution is correct, however, it falls right into the intended scope of ranking theory, and thereby turns Goodman's paradox into an unsolved riddle for Spohn's ranking theory.

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[42] For similar remarks, see also Spohn 2014, 113.

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